2nd Excellently LaTeX'ed Sample MOSP Orgy

Day I   8:00 AM – 12:30 PM

June 15, 2013

Note: For *any* problem, the first page of the solution must be a large, in-scale, clearly labeled diagram made with drawing instruments (ruler, compass, protractor, graph paper, carbon paper). Failure to meet any of these requirements will result in an automatic 0 for that problem.

1. Let a_1, a_2, …, a_9 be nine real numbers, not necessarily distinct, with average m. Let A denote the number of triples 1 <= i < j < k <= 9 for which $a_i + a_j + a_k$ >= 3m. What is the minimum possible value of A?

2. Let a,b,c be positive reals satisfying a+b+c=a^(1/7)+b^(1/7)+c^(1/7). Prove that a^a * b^b * c^c >= 1.

3. Let m_1, m_2, …, m_{2013} > 1 be 2013 pairwise relatively prime positive integers and A_1, A_2, …, A_{2013} be 2013 (possibly empty) sets with A_i a subset of {1,2,…,m_i-1} for i=1,2,…,2013. Prove that there is a positive integer N such that

N <= (2|A_1|+1) (2|A_2|+1) … (2|A_{2013}|+1)

and for each i=1,2,...,2013, there does <u>not</u> exist a in A_i such that m_i divides N – a.

S1. Let S be an s_0-dimensional projective algebraic variety, smooth or with mild singularities, and let S* be an ample divisor on S. Consider the canonical class S' of S.

(a) Prove that for any s >= s_0 + 1, sS* + S' is basepoint-free.

(b) Prove that for any s >= s_0 + 2, sS* + S' is very ample.

S2. Determine if there are nonzero complex numbers s_1, s_2, . . . , s_{2013} (not necessarily distinct) for which (s-s_1) * (s-s_2) * … * (s-s_{2013}) is a polynomial (in s) with integer coefficients, and

1 < prod (S=1 to 2013) of max(1, |s_S|) < 1.17628018.

S3. Prove that 10 is a primitive root modulo infinitely many primes.

Time: 4.5 hours
Each problem is worth 21 push-ups

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Day II 8:00 AM – 12:30 PM

June 16, 2013

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4. Triangle ABC is inscribed in circle w. A circle with chord BC intersects segments AB and AC at S and R, respectively. Segments BR and CS meet at L, and rays LR and LS intersect w at D and E, respectively. The internal angle bisector of angle BDE meets line ER at K. Prove that if BE = BR, then angle ELK = 1/2 angle BCD.

5. For what polynomials P(n) with integer coefficients can a positive integer be assigned to every lattice point in R^3 so that for every integer n >= 1, the sum of the n^3 integers assigned to any n x n x n grid of lattice points is divisible by P(n)?

6. Consider a function f: Z->Z such that for every integer n >= 0, there are at most $0.001n^2$ pairs of integers (x,y) for which f(x+y) =/= f(x) + f(y) and max |x|, |y| <= n. Is it possible that for some integer n >= 0, there are more than n integers a such that f(a) =/= a * f(1) and |a| <= n?

S4. Given a graph with labeled edges, the *stability* of a vertex is defined as the sum of the labels of all edges to which that vertex is incident. Determine whether there exists a tree with s > 2 edges such that, for every labeling of the edges with the numbers 1, 2, . . ., s (each used exactly once), there exist two vertices with the same stability.

S5. Find all integers s such that there exist positive integers s_1, s_2, s_3 satisfying

$$4/s = 1/s_1 + 1/s_2 + 1/s_3.$$

S6. Is pi^sqrt2 rational?

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